

PEPERIKSAAN PERCUBAAN SPM
SESI AKADEMIK 2023/2024
MATEMATIK TAMBAHAN

KERTAS 1
NOV

3472/1

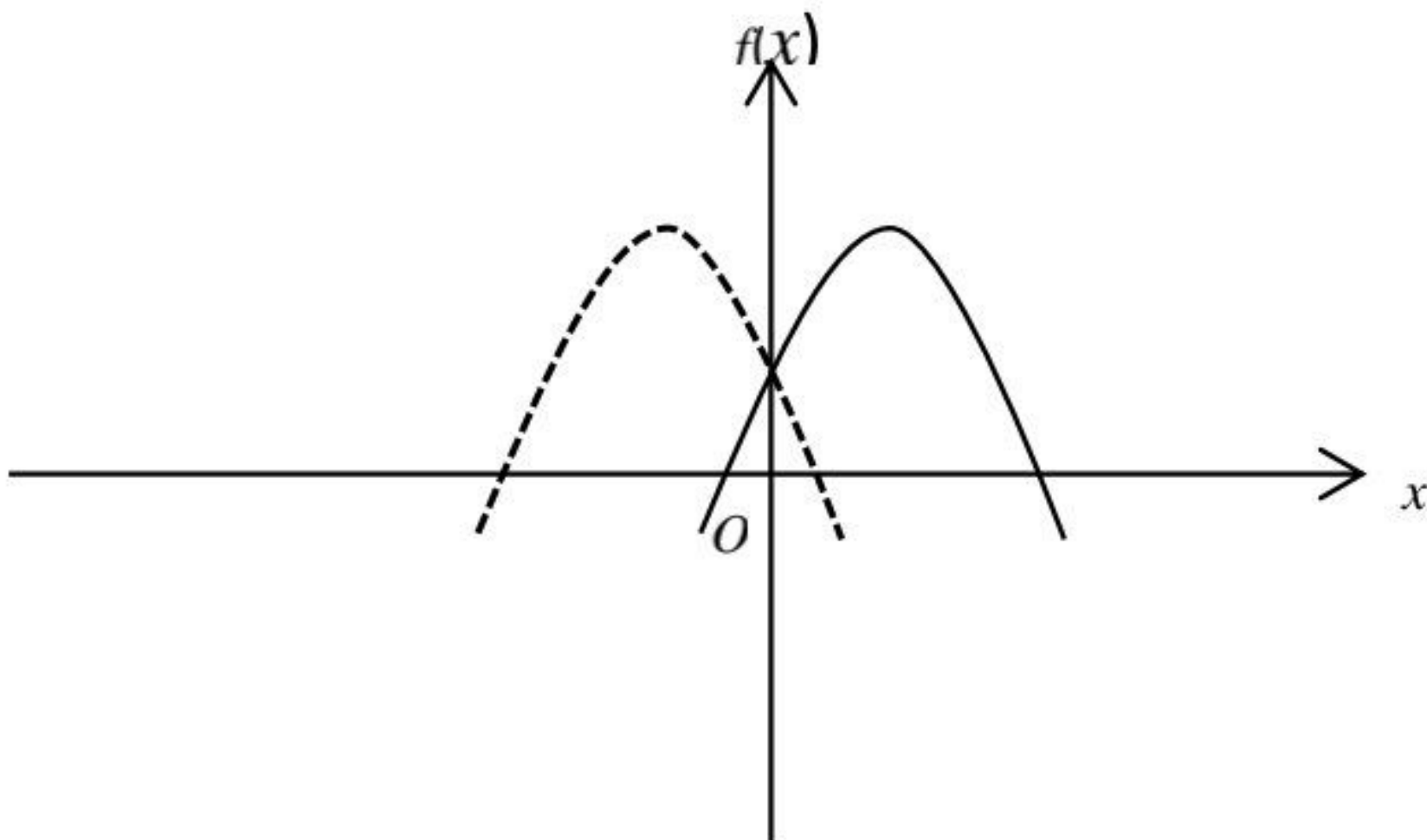
Dua jam

PANDUAN PENSKORAN

Panduan penskoran ini mengandungi 5 halaman bercetak.

PANDUAN PENSKORAN SET 1

Bahagian A

No.	Penyelesaian	Sub Markah	Jumlah
1	(a) $x = \frac{-(-5)}{2(-2)}$ or $f(x) = -2\left[x^2 + \frac{5}{2}x + \left(\frac{\frac{5}{2}}{2}\right)^2 - \left(\frac{\frac{5}{2}}{2}\right)^2 - \frac{c}{2}\right]$ $f(x) = -2\left[\left(x + \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 - \frac{c}{2}\right]$ $f(x) = -2\left(x + \frac{5}{4}\right)^2 + \left(\frac{25}{8}\right) + c$ $\frac{-4+b}{2} = \frac{-5}{4}$ $b = \frac{3}{2}$ $0 = -2(-4)^2 - 5(-4) + c$ $c = 12$ (b) $x = -\frac{5}{4}$ $\left(-\frac{5}{4}, \frac{121}{8}\right)$ (c) graf akan dipantulkan pada paksi y, tanpa mengubah bentuk graf.  Graf maksimum beranjak ke kanan	1 1 1 1 1 1 1	7

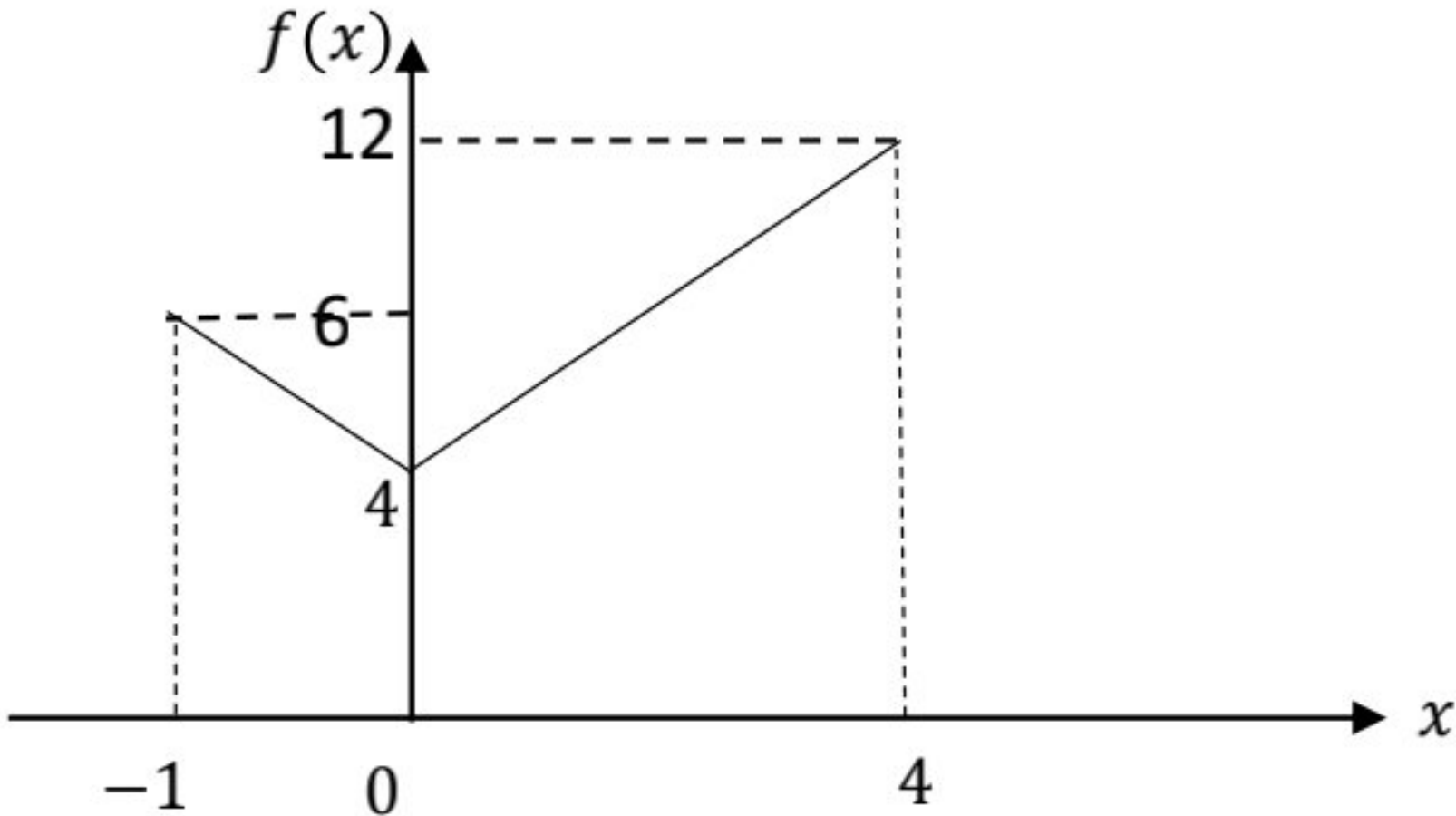
2	(a) $\frac{3^{2n+1}}{3^2} = 3^{2(2n)}$ $3^{2n+1-2} = 3^{4n}$ $2n + 1 - 2 = 4n$ $2n = -1$ $n = -\frac{1}{2}$ (b) i. $\log_{10} 2^{2x+1} = \log_{10} 12$ $(2x + 1) \log_{10} 2 = \log_{10} 12$ $2x + 1 = \frac{\log_{10} 12}{\log_{10} 2}$ $2x + 1 = 3.5850$ $2x = 2.5850$ $x = 1.29$ (ii) $\ln \frac{5\sqrt{3}}{2} = \log_e 5 + \log_e \sqrt{3} - \log_e 2$ $= \log_e 5 + \log_e 3^{\frac{1}{2}} - \log_e 2$ $= r + \frac{1}{2}q - p$	1 1 1 1 1 1 1	8
3	(a) $r = \frac{2}{3}$ (b) $45 = \frac{6k}{1 - \frac{2}{3}}$ $k = 2.5$ $6 \left(\frac{5}{2}\right) \left(\frac{2}{3}\right)^{n-1} < 1$ Gunakan logaritma $n = 8$	1 1 1 1 1	5

4	(a)	$xy = \frac{h}{p}x^2 + k$	1	4
	(b)	$k = 7$	1	
		$\frac{h}{p} = \frac{7 - 5}{0 - 3}$	1	
		$h = -\frac{2}{3}p$	1	

No.	Penyelesaian	Sub Markah	Jumlah
5	$\frac{1}{2} \left \begin{matrix} 2 & 6 & x & 2 \\ -2 & 7 & 0 & -2 \end{matrix} \right = 14$		3
	Or $ 14 - 2x + 12 - x = 28$	1	
	$ -3x + 26 = 28$		
	$-3x + 26 = \pm 28$	1	
	$-3x + 26 = 28 \quad , \quad -3x + 26 = -28$ $-3x = 28 - 26 \qquad \qquad -3x = 28 + 26$ $x = -\frac{2}{3} \qquad \qquad \qquad x = 18$		
	Maka, koordinat T($-\frac{2}{3}$, 0) atau T(18 , 0)	1	

6	(a) Vektor kedudukan zarah $A = -4\mathbf{i} + 5\mathbf{j} + t(2\mathbf{i} - \mathbf{j})$ atau Vektor kedudukan zarah $B = -8\mathbf{i} + 9\mathbf{j} + t(3\mathbf{i} - 2\mathbf{j})$ $-4\mathbf{i} + 5\mathbf{j} + t(2\mathbf{i} - \mathbf{j}) = -8\mathbf{i} + 9\mathbf{j} + t(3\mathbf{i} - 2\mathbf{j})$ $(2t - 4)\mathbf{i} + (5 - t)\mathbf{j} = (3t - 8)\mathbf{i} + (9 - 2t)\mathbf{j}$ $2t - 4 = 3t - 8 \quad \text{atau} \quad 5 - t = 9 - 2t$ $t = 4$ (b) Kedudukan zarah $A = -4\mathbf{i} + 5\mathbf{j} + 4(2\mathbf{i} - \mathbf{j})$ atau Kedudukan zarah $B = -8\mathbf{i} + 9\mathbf{j} + 4(3\mathbf{i} - 2\mathbf{j})$ $4\mathbf{i} + \mathbf{j}$ $(4, 1)$	1 1 1 1 1	6
7	$j + j + j\theta = 30$ $\frac{1}{2}j^2\theta = 50$ $\frac{1}{2}j(30 - 2j) = 50 \text{ atau setara}$ $(j - 10)(j - 5) = 0$ $j = 10 \text{ and } j = 5$ $\theta = 4 \text{ rad and } \theta = 1 \text{ rad}$	1 1 1 1	4
8	(a) (i) 4 (ii) had $f(x)$ tidak wujud, sebab had kiri tidak sama dengan had kanan. (b) (i) 2 (ii) 3	1 1 1 1	4

9	(a) -7 (b) $\int_4^6 [mx - q(x)] dx = 21$ $\int_4^6 mx dx - \int_4^6 q(x) dx = 21$ $\left[\frac{mx^2}{2} \right]_4^6 - 7 = 21$ $\frac{m(6)^2}{2} - \frac{m(4)^2}{2} = 28$ $m = \frac{14}{5}$	1 1 1 1	4
10	(a)(i) Bilangan cara: $P = (3 - 1)! \times 3!$ $= 12$ (ii) $\frac{{}^3P_1 \times 6! \times {}^5P_1}{2! 2! 2!} @ \frac{3 \times 6! \times 5}{2! 2! 2!}$ $= 1350$ (b) ${}^5C_2 \times {}^3C_1$ ${}^3C_2 \times {}^5C_1$ ${}^5C_2 \times {}^3C_1 + {}^3C_2 \times {}^5C_1 @ 30 + 15$ 45	1 1 1 1 1 1 1	8
11	(a) (i) $\mu = \frac{50+60}{2}$ $\mu = 55$ (ii) $\sigma_1 > \sigma$, serakan taburan normal lebih besar daripada nilai min, μ dan ketinggian graf berkurang. (b) (i) $P(55 < X < 70) = 0.68 + \frac{0.95-0.68}{2}$ $= 0.68 + 0.135$ $= 0.815$	1 1 1	5

	(ii) $P(X < 50) = \frac{1.00 - 0.95}{2}$ $= \frac{0.05}{2}$ $= 0.025$ (iii) $P(X > 75) = \frac{1 - 0.998}{2}$ $= \frac{0.002}{2}$ $= 0.001$	1 1	
12	(a) (i) $x = 1, -1$ (ii) $f(4) = 2(4) = 8$ $0 \leq f(x) \leq 8$ (b)  $4 \leq f(x) \leq 12$	1 1 1 1 1 1	6

Bahagian B

No.	Penyelesaian	Sub Markah	Jumlah
13	(a) $f^{-1}(x)$ bukan fungsi. (b) $x \leq -1$ @ $x \geq 1$ (c) Domain $0 \leq x \leq 7$ Julat $0 \leq g^{-1}(x) \leq 5$	2 1 1 2 1 1	4

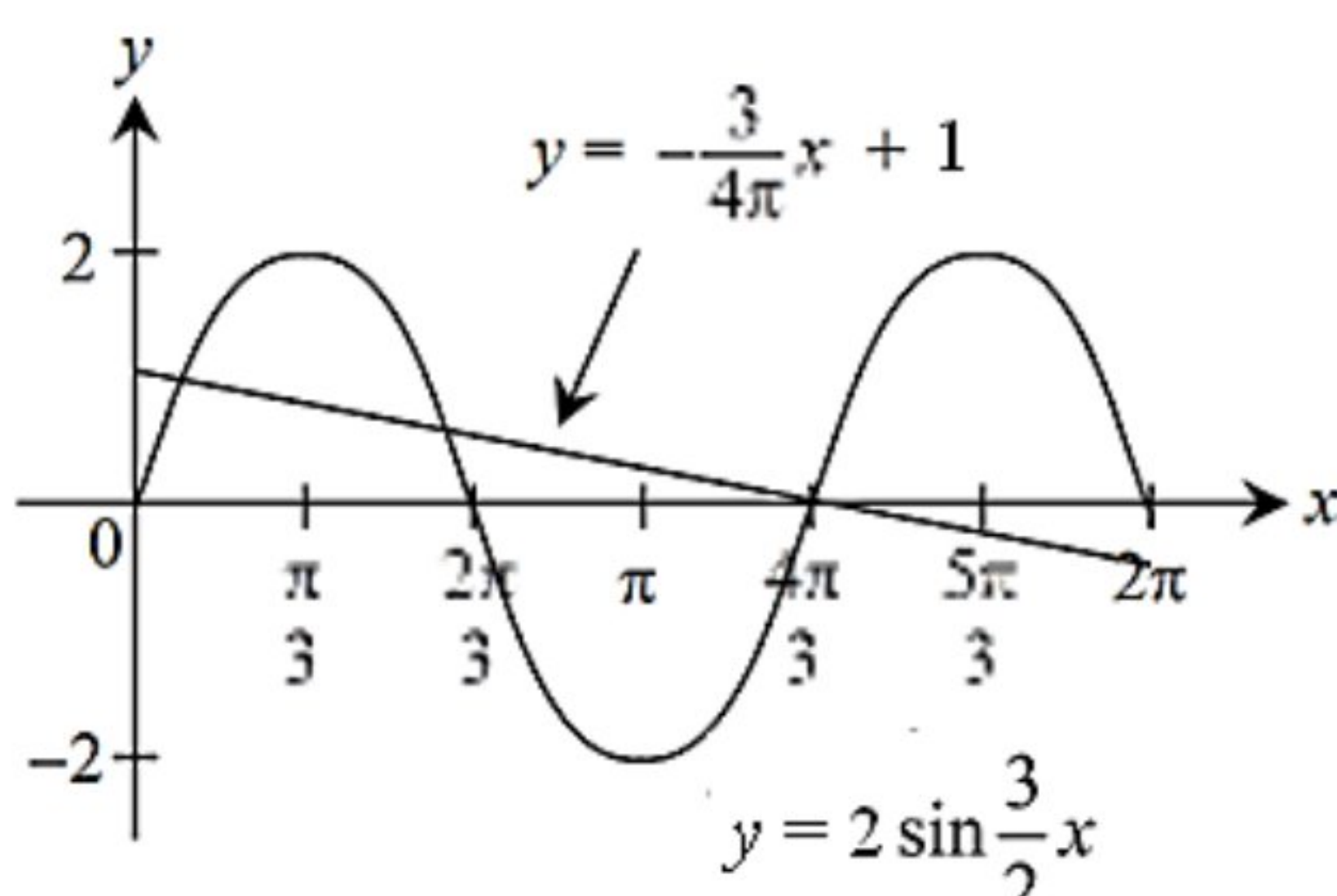
14	(a) $\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \left(\frac{2 - \sqrt{3}}{2 - \sqrt{3}} \right) + \frac{12}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$ $= \frac{4 - 2\sqrt{3} - 2\sqrt{3} + 3}{4 + 2\sqrt{3} - 2\sqrt{3} - 3} + \frac{12\sqrt{3}}{3}$ $= \frac{7 - 4\sqrt{3} + 4\sqrt{3}}{1} = 7$ (b) $2(3 + 5\sqrt{7}) \times (3 + 5\sqrt{7})$ $= (6 + 10\sqrt{7}) \times (3 + 5\sqrt{7})$ $= 18 + 30\sqrt{7} + 30\sqrt{7} + 350$ $= 368 + 60\sqrt{7}m^2$ a = 368; b = 60	1 1 1 1 1 1 1,1	8
15	(a) (i) $2 \sin \theta \cos \theta$ $2 \left(\frac{p}{\sqrt{1+p^2}} \right) \left(\frac{1}{\sqrt{1+p^2}} \right)$ $\frac{2p}{1+p^2}$ (ii) $2 \cos^2 \frac{\theta}{2} - 1 = \cos \theta$ $2 \cos^2 \frac{\theta}{2} = \cos \theta + 1$ $2 \cos^2 \frac{\theta}{2} = \frac{1}{\sqrt{1+p^2}} + 1$ $\cos^2 \frac{\theta}{2} = \frac{1 + \sqrt{1+p^2}}{2\sqrt{1+p^2}}$ (b) $2 \cos^2 \theta - 1 = 5 \cos \theta - 3$ $2 \cos^2 \theta - 5 \cos \theta + 2 = 0$ $(\cos \theta - 2)(2 \cos \theta - 1) = 0$ $\cos \theta = 2 \text{ (tiada penyelesaian) or } 2 \cos \theta = 1$ $\cos \theta = \frac{1}{2}$	1 1 1 1 1 1	5

		1	
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PANDUAN PEMARKAHAN KERTAS 2 - SET 1

No.	Skema Pemarkahan		Sub Markah	Jumlah Markah
1	$x + 3y + 2z = 9$ @ $2x + y + 3z = 9$ @ $3x + 4y + z = 14$		1	7
	Hapus anu pertama dengan kaedah penggantian/ penghapusan		1	
	$5y + z = 9$ @ $5y + 5z = 13$		1	
	Hapus anu kedua dengan kaedah penggantian/ penghapusan		1	
	$4z = z$		1	
	$z = 1$		1	
	$x = 2.2$		1	
2	$y = 1.6$		1	6
	Pen = RM2.20, Pensel = RM1.60, Pembaris = RM1.00		1	
	(a)	$f(x) = x^2 + 2kx + \left(\frac{2k}{2}\right)^2 - \left(\frac{2k}{2}\right)^2 + 5k - 3$	1	
		$f(x) = (x + k)^2 - k^2 + 5k - 3$	1	
		$-k^2 + 5k - 3 = -17$	1	
		$k = 7, k = -2$	1	
	(b)	$f(x) = -(x - 2)^2 + 17$	1	
		titik maksimum = (2,17)	1	
3	(a)	$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}$ atau $\overrightarrow{PB} = \overrightarrow{PA} + \overrightarrow{AB}$ atau $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ atau setara (guna hukum Δ)	1	7
		(i) $\overrightarrow{PB} = -\frac{2}{3}(9\underline{x}) + 4\underline{y}$ @ $\overrightarrow{PB} = \frac{1}{3}(9\underline{x}) - 9\underline{x} + 4\underline{y}$	1	
		$\overrightarrow{PB} = -6\underline{x} + 4\underline{y}$	1	
	(b)	(ii) $\overrightarrow{OQ} = 9\underline{x} + \frac{1}{2}(-9\underline{x} + 4\underline{y})$	1	
		$\overrightarrow{OQ} = \frac{9}{2}\underline{x} + 2\underline{y}$	1	
		$\overrightarrow{PR} = -\frac{12}{5}\underline{x} + \frac{8}{5}\underline{y}$ atau setara	1	
		$\overrightarrow{PR} = \lambda \overrightarrow{PB}$	1	
		$-\frac{12}{5}\underline{x} + \frac{8}{5}\underline{y} = \lambda(-6\underline{x} + 4\underline{y})$	1	4
		Bandingkan pekali $\underline{x}$ dan $\underline{y}$	1	
		$-\frac{12}{5} = -6\lambda$ dan $\frac{8}{5} = 4\lambda$	1	
		$\lambda = \frac{2}{5}$ atau setara	1	
			1	

4	(a)	$\frac{dy}{dx} = 2x - 3$ $\frac{dy}{dx} = 2(1) - 3$ $\frac{dy}{dx} = -1$ $y - 2 = -1(x - 1)$ $y = -x + 3$	1		
			1		
			1	3	
	(b)	$x = 3, \quad \frac{dy}{dx} = 3$ kecerunan normal, $m = -\frac{1}{3}$ $y - 4 = -\frac{1}{3}(x - 3)$ $y = -\frac{1}{3}x + 5$	1		
			1	2	
	(c)	Selesaikan persamaan serentak $-x + 3 = -\frac{1}{3}x + 5$ $x = 3$ $y = -(-3) + 3$ $y = 6$ $C(-3, 6)$	1		
			1		
			1	3	8
5	(a)	$RHS : \frac{\sin^2 \theta}{\sin^2 \theta} + 1$ $LHS : \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$	1		
		$\frac{1}{\cos^2 \theta} \times \frac{\sin^2 \theta}{\sin^2 \theta}$ $\frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{1}{\sin^2 \theta}$	1		
		$\frac{\tan^2 \theta}{\sin^2 \theta}, LHS \text{ dibuktikan.}$	1	3	

	(b)	 bentuk graf sinus 1 $\frac{3}{2}$ kitaran pada 2π 1 Amplitud = 2 1 Persamaan garis lurus, $y = -\frac{3}{4\pi}x + 1$ 1 Bilangan penyelesaian 1	5	8
6	(a)	$\frac{1}{2}(9)^2 \left(90 \times \frac{3.142}{180} \right)$ 63.63 1 1	2	6
	(b)	$\angle ROS = \pi - \left(45 \times \frac{3.142}{180} \right)$ 63.63 1 1 $S_{RS} = 9(2.3565)$ atau $RS^2 = 9^2 + 9^2 - 2(9)(9) \cos(135^\circ)$ atau $S_{PQ} = 9 \left(90^\circ \times \frac{3.142}{180^\circ} \right)$ 1 $21.21 + 16.63 + 14.14 + 9 + 9$ 1 69.98 1	4	
7	(a)	$a(3)^2 + b(3) = 51$ $3a + b = 17$ atau $a(8)^2 + b(8) = 296$ 1 $8a + b = 37$ Selesaikan persamaan serentak $17 - 3a = 37 - 8a$ 1 $a = 4$ 1 $b = 5$ 1	4	6
	(b)	$T_1 = S_1 = 4(1)^2 + 5(1)$ 1 $T_2 = 4(2)^2 + 5(2) - 9$ $d = 17 - 9$ $d = 8$ 1	2	

8	(a)	$y = \frac{2x^2}{2} + c$ $y = x^2 + c$ $K(3,12)$ $12 = 3^2 + c$ $c = 3$ $y = x^2 + 3$	1		
			1		
			1	3	
	(b)	<i>Luas trapezium</i> $\frac{1}{2}(3)(3+12)$ <i>Luas di bawah graf</i> $\int_0^3 x^2 + 3 dx$ $\left[\frac{x^3}{3} + 3x\right]_0^3$ $\left[\left(\frac{3^3}{3} + 3(3)\right) - \left(\frac{0^3}{3} + 3(0)\right)\right]$ <i>Luas kawasan berlorek</i> $= \frac{45}{2} - \left[\left(\frac{3^3}{3} + 3(3)\right) - \left(\frac{0^3}{3} + 3(0)\right)\right]$ $= \frac{9}{2}$	1		
			1		
			1	4	
	(c)	$= \pi \left[\frac{y^2}{2} - 3y\right]_3^t$ $= \pi \left[\left(\frac{t^2}{2} - 3t\right) - \left(\frac{3^2}{2} - 3(3)\right)\right]$ $= \pi \left(\frac{t^2}{2} - 3t + \frac{9}{2}\right)$	1		
			1		
			1	3	10
9	(a)	(i)	$2.8 = 10p$ $p = 0.28$ <i>sisihan piawai</i> $= \sqrt{10(0.28)(1-0.28)}$ 1.4200	1	
				1	
		(ii)	$P(X=0) + P(X=1) + P(X=2)$ ${}^{10}C_0 (0.28)^0 (0.72)^{10} + {}^{10}C_1 (0.28)^1 (0.72)^9 +$ ${}^{10}C_2 (0.28)^2 (0.72)^8$ 0.4378 $P(-1.4231) - P(-1.8077)$ 0.04202	1	
				1	
				1	
				1	
	(b)	(i)	$P(z > 0) - P(z > k) = 0.2806$		
				5	

		$0.5 - 0.2806$ 0.2194 (ii) $P(-k < z < k)$ $1 - P(z > k) - P(z < -k)$ $1 - 2P(z > k)$ $1 - 2(0.2806)$ 0.5612	1 1 1 1 1	5	10														
10	(a)	(i) $\frac{6}{-h} = -1$ $h = 6$ (ii) $T = \left(\frac{3(6) + 1(-2)}{3 + 1}, \frac{3(0) + 1(4)}{3 + 1} \right)$ $T = (4, 1)$ (iii) $\frac{1}{2} \begin{vmatrix} 4 & -2 & -4 & 4 \\ 1 & 4 & -2 & 1 \end{vmatrix}$ dan $\frac{1}{2} (4(4) + (-2)(-2) + (-4)1) - (4(-2) + (-4)4 + (-2)1) $ $\frac{1}{2} 42 $ 21	1 1 1 1 1 1 1	7	10														
	(b)	$\left(\frac{y - 4}{x + 2} \right) \left(\frac{y}{x - 6} \right) = -1$ $y^2 - 4y = -x^2 + 4x + 12$ $x^2 + y^2 - 4x - 4y - 12 = 0$	1 1 1	3															
11	(a)	<table><tr><td>$\log_{10} x$</td><td>1.0</td><td>1.68</td><td>2.12</td><td>2.75</td><td>3.01</td><td>3.50</td></tr><tr><td>$\log_{10} y$</td><td>2.15</td><td>2.75</td><td>3.25</td><td>3.83</td><td>4.08</td><td>4.53</td></tr></table>	$\log_{10} x$	1.0	1.68	2.12	2.75	3.01	3.50	$\log_{10} y$	2.15	2.75	3.25	3.83	4.08	4.53	1 1	2	10
$\log_{10} x$	1.0	1.68	2.12	2.75	3.01	3.50													
$\log_{10} y$	2.15	2.75	3.25	3.83	4.08	4.53													
	(b)	(rujuk graf pada muka surat 9) Plot $\log_{10} y$ melawan $\log_{10} x$ dengan paksi dan skala seragam Salah satu titik diplot dengan betul Semua titik diplot dengan betul Garis lurus penyesuaian terbaik	1 1 1 1	3															
	(c)	(i) $\log_{10} y = (2b + 1)\log_{10} x + \log_{10} a$ (ii) $\log_{10} a = 1.12$ $a = 14.96$ (iii) $2b + 1 = \frac{4.53 - 3.83}{3.5 - 2.75}$ $2b = -\frac{1}{15}$ $b = -\frac{1}{30}$	1 1 1 1 1	5															

12	(a)	(i)	$\angle BCD = 86^\circ$ $Luas BCD = \frac{1}{2}(10.4)(16.9) \sin 86^\circ$ 87.67	1 1 1	7	10
		(ii)	$BD^2 = 10.4^2 + 16.9^2 - (10.4)(16.9) \cos 86^\circ$ $BD = 19.22$	1 1		
		(iii)	Katakan jarak terdekat dari titik C ke BD ialah t $\frac{1}{2}(t)(19.22) = 87.67$ $t = 9.123$	1 1		
	(b)	$\frac{\sin \angle ABD}{15.1} = \frac{\sin 94^\circ}{19.22}$ $\angle ABD = 51.60^\circ$ $\angle ADB = 34.40^\circ$ $\frac{\sin \angle BDC}{16.9} = \frac{\sin 86^\circ}{19.22}$ $\angle BDC = 61.30^\circ$ $\angle ADC = 34.40^\circ + 61.30^\circ$ $= 95.70^\circ$		1 1 1	3	
13	(a)	$\frac{21}{p} \times 100 = 150$ $p = 14$ $\frac{24}{15} \times 100 = q$ $q = 160$ $\frac{r}{10} \times 100 = 150$ $r = 15$		1 1 3	3	
		(b)	$\frac{20}{14} \times 100 = 142.86$	1	1	
		(c)	$\frac{150(15) + 160(10) + 150(12) + 70(13)}{15 + 10 + 12 + 13}$ 131.20	1 1	2	
	(d)	$A : I_{\frac{22}{16}} = \frac{130 \times 150}{100}$ 195 $D : I_{\frac{22}{16}} = \frac{70 \times 85}{100}$ 59.50 $\frac{195(15) + 160(10) + 150(12) + 13(59.5)}{15 + 10 + 12 + 13}$		1 1 1		

		141.97	1	4	10	
14	(a)	I) $x + y \leq 10$	1	3	10	
		II) $3x + 2y \leq 24$	1			
	III) $y \leq 2x$	1	3			
	(b)	(rujuk graf pada muka surat 8)		1		3
		Lukis dengan betul sekurang-kurangnya satu garis lurus daripada *ketaksamaan melibatkan x dan y				
Lukis dengan betul semua *garisan lurus daripada *ketaksamaan melibatkan x dan y		1				
(c)	(i)	$2 \leq x \leq 5.4$	1	4	10	
	(ii)	$Keuntungan = 280x + 200y$ $Keuntungan maksimum = 240(4) + 200(6)$ 2320	1 1 1			
15	(a)	$v = 3 + 2t - t^2$	1	2	10	
		$a = \frac{dv}{dt} = 2 - 2t$				
	Apabila $a = 0$	1	2			
	$2 - 2t = 0$					
	$t = 1 \text{ s}$					
(b)	$V_{maks} = +2(1) - (1)^2$	1	2			
	$= 4$	1				
(c)	$v = 0$	1	4			
	$3 + 2t - t^2 = 0$					
	$t^2 - 2t - 3 = 0$					
	$(t - 3)(t + 1) = 0$					
$\therefore t = 3$	(mesti tunjukkan pemfaktoran)	1				
$s = \int 3 + 2t - t^2 dt$		1				
$= 3t + t^2 - \frac{t^3}{3} + c$						
$t = 0, s = 0, c = 0$		1				
$s = 3t + t^2 - \frac{t^3}{3}$						
Berhenti di G, $t = 3 \text{ s}$		1				
$s = 3(3) + (3)^2 - \frac{(3)^3}{3}$						
9		1	4			
		1				
(d)	Kereta Syafiq	1	2	10		
	$t = 3, v = -4, s = -4t$					
$s = -4(3) = -12 \text{ cm}$		1	2	10		
$Jarak = 40 - 9 - 12$						
19		1	2	10		
		1				

